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On Mr. Kielkopf's Not So Sober Understanding of Standard Elementary Logic

ROBERTO TORRETTI

In his recent article 'The Binary Operation called "Material Implication" Soberly Understood' (*Mind,* lxxxi (1972), 338–347), Charles F. Kielkopf presents three cases of arguments valid within standard elementary logic, where true premisses allegedly lead to a false conclusion. This, he suggests, should induce us to burn all the familiar textbooks of logic, for, in the light of these examples, they appear to contain 'considerable portions of sophistry and illusion' (p. 338). Fortunately, he adds, a less inflammatory solution is readily available. It consists essentially in the following: proofs of validity supplied by standard elementary logic must be understood as proofs of relative validity; if q is a consequence under elementary logic of premiss p, and p is true, all we thereby know about q is that it cannot be falsified through certain procedures relevant to elementary logic—but it might very well be falsified through other procedures.

A simple consideration will persuade us that Kielkopf's solution is indeed a conservative one—since it enables us to retain not only standard logic but a good deal else besides—but that, nevertheless, it is quite radical. Take any arbitrary physical hypothesis which is not directly verifiable by observation (in a very literal sense of direct verification); call it p. Suppose an empirically false sentence s is a logical consequence (by elementary logic) of our hypothesis p and the accepted truths q_1, q_2, . . . q_r. This means that elementary logic enables us to infer the truth of not-p from the empirically true sentence not-s and the accepted truths q_1, q_2, . . . q_r. This would have sufficed in the old days to reject hypothesis p (unless we were willing to tamper with accepted truths). But it need no longer be so after Kielkopf's conservative revolution: The truth of the premisses not-s, q_1, q_2, . . . q_r guarantees that we cannot falsify not-p by any procedure relevant to standard elementary logic. But could we not falsify it eventually by some other procedure? Not until all such procedures have been tried, not until every developed or undeveloped branch of logic has been exhaustively examined, can we be sure that not-p is true, given that not-s, q_1, q_2, . . . q_r are true. We may therefore entertain the wildest hypotheses until 'we have surveyed all methods for determining truth', which, as Kielkopf himself surmises on page 347, could very well not happen before the Greek Calends.

The epistemological impact of Kielkopf's reform of logical theory is bound to be tremendous. Unfortunately, I find that his problems admit a less inspiring, indeed a rather trivial solution. In the following I assume that 'standard logic' or 'elementary logic' in Kielkopf's parlance is the first order predicate calculus with the standard syntactical and semantical rules (for the latter see, e.g. Schoenfield, *Mathematical Logic*, Reading, Mass., 1967, pp. 18–19, whose terminology I shall use).

Kielkopf's first problem is this: Both $\sim(\exists x)Ux \supset (x)(Ux \supset Hx)$ and $\sim(\exists x)Ux \supset (x)(Ux \supset \sim Hx)$ represent logical truths in standard elementary logic. Therefore, I may validly infer, by *modus ponens*, from $\sim(\exists x)Ux$, both $(x)(Ux \supset Hx)$ and $(x)(Ux \supset \sim Hx)$. But if Ux is taken to mean 'x is an unicorn' and Hx is taken to mean 'x has one horn', the premiss and the first of these conclusions are true but the second conclusion is false. Kielkopf writes: 'I can admit that if I restricted myself to the inspection of unicorns, it would be impossible to refute "No unicorns have one horn". But to admit this does not preclude me from turning to fables where I find evidence for rejecting "no unicorns have one horn" while still accepting that there are no unicorns' (p. 341). I understand this passage as an informal allusion to so-called free logic (a very readable introduction to which may be found in Lambert and van Fraassen, *Derivation and Counterexample*, Encino, Calif., 1972). It should be clear, however, that the formulae of standard elementary logic are not the formulae of free logic; even if the syntactical rules happen to be the same in both logics, the semantical rules fixing the logical significance of the formulae definitely are not the same. Now, under the standard semantical rules of elementary logic, if $(x)(Ux \supset \sim Hx)$ is false in a given structure, $(\exists x)\sim(Ux \supset \sim Hx)$, $(\exists x)(Ux \;\&\; Hx)$ and $(\exists x)Ux$ must all be true in that structure. Consequently, there cannot exist a structure for the lower predicate calculus in which $\sim(\exists x)Ux$ is true and $(x)(Ux \supset \sim Hx)$ is false, and Kielkopf's 'translation' of these formulae, if it truly presupposes that such a structure exists, is simply an inadmissible 'translation'.

Kielkopf's third problem may be formulated thus: $\sim(p \supset q) \supset (q \supset p)$ is a tautology (indeed, it is easily derived from $(\sim p \vee q) \vee (\sim q \vee p)$); therefore, the following sentence is logically true in standard logic: 'If it is not the case that if Molly has seen a gorilla during the third month of pregnancy then Molly's baby will look like a hairy ape, then if Molly's baby will look like a hairy ape then Molly has seen a gorilla during the third month of pregnancy.' The solution of this problem is so familiar that it should not be necessary to repeat it here. The 'If . . . then . . .' construction in ordinary English is not governed by a truth-functional semantic rule (i.e. the truth-value of the sentence so constructed does not depend functionally on the truth-values of the sentences filling the blanks of the 'If . . . then . . .' schema). I would venture to add that the rule governing this construction, when it is applied to untensed indicative sentences with well-defined truth-values, is a partial truth-function, defined for arguments $\langle T,T\rangle$, $\langle T,F\rangle$, and $\langle F,F\rangle$, but undefined for argument $\langle F,T\rangle$. ('If *this* is a correct application of existential instantiation, then I am Lord Russell's bald King of France' is a [true] sentence we may at times feel inclined to say to a student; on the other hand, the following seems to me to be neither true nor false, but bordering on nonsense: 'If Pablo Picasso is a celebrated German obstetrician, then Chairman Mao is a well-known Chinese chairman'.) The truth-function governing what Kielkopf calls 'material implication' would then be an extension of this partial function to the full domain of $\{T,F\}^2$. Because of this, 'If p then q' can represent the meaning of $p \supset q$ fairly well in certain contexts.

But in other contexts, as every teacher of elementary logic knows, this sort of 'translation into ordinary English' can be notoriously perverse. In his third problem, Kielkopf has produced a remarkable example of the second kind of context.

I have left the remaining difficulty for the end, because I fail to see in it any problem at all. Kielkopf quotes the tautology $\sim p \supset (p \supset q)$ (an obvious equivalent of $(p \vee \sim p) \vee q$) and 'translates' it thus: 'If I am not in California, then if I am in California, I am the governor of California.' This sentence is indeed rather stupid, and I cannot imagine a practical situation where I could use it straight-forwardly as it stands, but I would not say it is false or *stricto sensu* nonsensical, and I would be ready to quote it, say, in a philosophical conversation, as an example of the awkwardness of some truisms.

My solution of Kielkopf's first and last problems involves a restriction in the scope of elementary logic, since it appeals to the fact that not every ordinary English sentence can be non-trivially represented by a closed formula of the lower predicate calculus (the trivial representation of any sentence by a sentence parameter is always possible, if you embed the propositional calculus in the lower predicate calculus after the manner of Benson Mates—see his *Elementary Logic*, New York, 1965). This restriction, however, is no novelty, and no one that I know has ever thought differently on this point. (There was of course a time when it was fashionable to maintain that those sentences which could not be represented in the predicate calculus of order ω lacked 'cognitive significance'; but that is quite another matter.) I believe also—but this is no part of my solution —that those sentences which cannot be represented in the lower predicate calculus can appear as premisses or conclusions of cogent arguments which could eventually be formalized in systems of logic beyond the pale of elementary standard logic. Such a belief is fairly common nowadays. But the possibility of these broader systems of logic does not imply, as Kielkopf appears to think, that the valid inferences of elementary logic could be invalidated within these new developments. If this were so, elementary logic would be of very little use, even if it assured us of what Kielkopf terms the 'relative validity' of some argument schemata.

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